Modeling Swell, High Frequency Spreading and Wave Breaking

V. E. Zakharov
Department of Mathematics, University of Arizona
Tucson, AZ 85721, USA
phone: (520) 621-4841 fax: (520) 621-8322 email: zakharov@math.arizona.edu

Award Number: N000149810070

LONG-TERM GOALS

My long-term goal is to contribute to understanding of the nonlinear dynamics of the wave sea surface excited by wind. Of particular interest to me is the role of such interaction in long-term prediction of wave amplitudes excited in rough seas.

OBJECTIVES

I wish to develop the phenomenological diffusion model of interaction of gravitational waves on water surface in presence of wind and viscosity. Motivation for development of such a model is the fact that numerical solvers based on Hasselmann kinetic equation for waves are time-consuming and hardly can be used for practical purposes. Numerical solver based on the diffusion model is expected to be at least three orders of magnitude faster.

APPROACH

I propose two generations of simplified model of Hasselmann kinetic equation for waves. The first generation model

$$\frac{\partial n}{\partial t} = \alpha L \omega^{24} n^3 \qquad (1)$$

where $n_k \delta(k-k') = < a(k)a(k') >$, $a(k)$ is the complex normal amplitude [1], $n_{-k} \neq n_k$, **L** is linear operator

$$L = \frac{1}{\omega^3}\left[\frac{1}{2}\frac{\partial^2}{\partial \omega^2} + \frac{1}{\omega^2}\frac{\partial}{\partial \varphi^2}\right] \qquad (2)$$

Eq.(1) is the diffusion approximation obtained from differential approximation to the kinetic equation [2], [3]. One should emphasize that equation (1) is the model equation and the value of the constant α can be defined only from comparison with the results of numerical simulation of the kinetic equation or laboratory measurements. The results of numerical simulation of the Eq.(1) have shown good agreement of angle averaged frequency directional distribution of the nonlinear interaction term with the corresponding results for kinetic equation (see [4], [5]). The correspondence of angular distributions for particular frequency values was not, however, very good.

The second generation diffusion model takes into account the effects of non-locality in the nonlinear interaction term while preserving such important properties of the first generation diffusion model as scaling, conservation laws and Kolmogorov solution:

$$\frac{\partial n}{\partial t} = L\omega^{24}\left(\alpha_1 n^3 + \alpha_2 n^2\langle n\rangle + \alpha_3 n\langle n^2\rangle + \alpha_4 n\langle n\rangle^2\right) \tag{3}$$

where $\langle n\rangle = \frac{1}{2}\int n d\omega d\varphi$ and $\langle n^2\rangle = \frac{1}{2}\int n^2 d\omega d\varphi$.

Besides the term $\alpha_1 n^3$ in the right-hand side that is similar to the first generation model there are the other terms representing all possible combination of the terms which are cubical with respect to n and contain an integration over ϕ. We intentionally omitted the term $\left(\int n d\varphi\right)^3$ and $\int n^3 d\varphi$ as they make no contribution into angular anisotropy. As in the case of first generation diffusion model, the coefficients $\alpha_i, i = 1{,}4$ are defined from the comparison with the results of numerical simulation of kinetic equation or experimental measurements. Obviously, the choice of the coefficients $\alpha_1 \neq 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$ corresponds to the first generation model.

The following is the description of the algorithm of definition of the set of coefficients $\alpha_i, i = 1{,}4$. Let $S_{nl}(n(\omega,\varphi))$ be nonlinear interaction term at the kinetic equation, $n_j = n_j(\omega,\varphi)$ be JONSWAP initial conditions [4], [5] and $S_{nl} = \sum_{i=1}^{4}\alpha_i S_i(n(\omega,\varphi))$ be nonlinear interaction term at the second generation model (3), where

$$S_1(n) = Ln^3, S_2(n) = Ln^2\langle n\rangle, S_3(n) = Ln\langle n^2\rangle, S_4(n) = Ln\langle n\rangle^2$$

We are looking for the best approximation to the function $S_{nl}(n_j)$ by the linear combination of the functions $S_i(n_J), i = 1{,}4$ in the norm L^2:

$$\frac{\partial A}{\partial \alpha_i} = 0, A(\alpha_1,\alpha_2,\alpha_3,\alpha_4) = \int \left(S_{nl}(n_J) - S(n_J)\right)^2 d\omega d\varphi \tag{4}$$

The coefficients $\alpha_i, i = 1{,}4$ are the solution of the system of equations (4) which is the system of linear equations

$$B_{ij}\alpha_{ij} = C_j, i, j = 1{,}4 \tag{5}$$

where symmetric matrix B_{ij} and vector C_{ij} are defined by

$$B_{ij} = \int S_i(n_J)S_j(n_J)d\omega d\varphi, C_j = \int S_{nl}(n_J)S_J(n_J)d\omega d\varphi, i, j = 1{,}4$$

Both the first and second generation diffusion models overestimated the nonlinear interaction in the case of narrow directional spectra. In FY2000, to fix this a third generation model was formulated in which there is an expansion about the wind direction. For very narrow angular spectra, this formulation should vanish as expected from the full calculation. Scaling coefficients must be found by comparison t the exact Hasselmann equation.

Additionally, we proposed a "clean test" for checking the accuracy of the various nonlinear approximation models. This calculation is based on control of the conservation constants; action, momentum and energy.

In a spearate action, a model for rogue waves was also proposed based on the integral equation containing the ouplinc coefficient for surface waves. By using a simple approximation for the coefficient based on the modulus of the interacting wave numbers it is possible to arrive a a quasi-soliton solution for the formation of localized wave groups. Numerical simulations in one dimension suggest that quasi-solitons can merge in non-elastic collisions generating waves of very high amplitude from initially smooth conditions.

The numerical part of the work was performed by Dr. Andrei Pouchkarev, Department of Mathematics, University of Arizona.

WORK COMPLETED

First and second generation diffusion model were proposed. Based on these models supplied by viscous and forcing terms, computer codes have been created. Both codes have been shown to reproduce important properties of the original Hasselmann equation -- stationary Kolmogorov solutions. Typical calculation of the evolution of the turbulence driven by external forcing from the initial low-level random noise conditions to the stationary equilibrium state on the grid of 40x130 nodes in angle-frequency space took the time of the order of dozens of minutes using Pentium-133 MHz PC. Comparison of the results of exact Hasselmann equation, first and second generation diffusion models has been made. It was shown what are the advantages of the second generation model with respect to the first generation one.

RESULTS

We calculated the coefficients $\alpha_i, i = 1,4$ as the solution of the system of linear equations (5) using the functions n_J and $S_{nl}(n_J)$ from numerical simulation of kinetic equation for waves by Resio and Tracy [4], [5]: $\alpha_1 = -2.35 \cdot 10^{-3}, \alpha_2 = 5.14 \cdot 10^{-2}, \alpha_3 = -9.76 \cdot 10^{-2}, \alpha_4 = 4.67 \cdot 10^{-1}$.
Both first and second generation model qualitatively correctly reflect the behavior of $S_{nl}(f,\varphi)$ in the kinetic equation. It should be noted that both of them fail to reproduce such fine details of the $S_{nl}(f,\varphi)$ distribution as two-hump maximum and small-amplitude high-frequency tail.

Fig.1 represents directional (angle averaged) distribution of the spectrum as a function of the frequency for three models. The correspondence of the second generation model with Resio-Tracy model is better than first generation model.

Fig.2 represents angular dependence at the frequency of the minimum of the spectrum $f=0.16$. Second generation model improves correspondence with Resio - Tracy model with respect to the first generation model.

Fig.3 represents nonlinear interaction term distribution as the function of the frequency at zero angle. While the second generation model dependence is much closer to Resio-Tracy results in the vicinity of the minimum and the maximum of the spectrum, it still fails to describe high - frequency tail of the distribution.

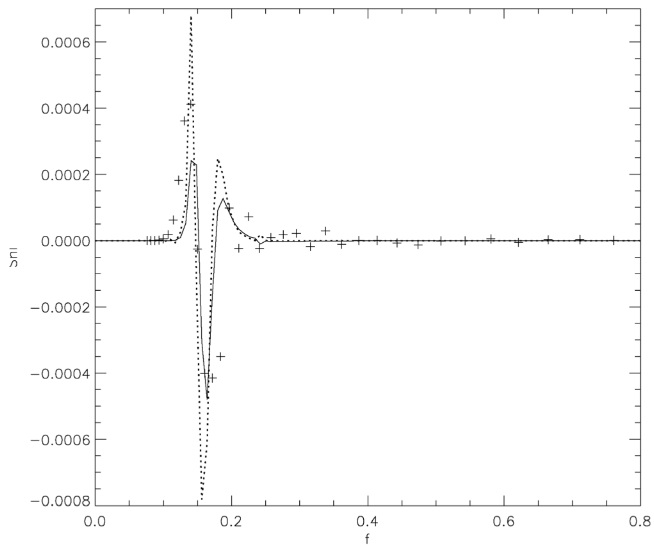

Figure 1. Directional (angle averaged) spectrum as a function of the frequency f. Crosses represent Resio-Tracy results, dashed line -- first generation model, solid line - second generation model.

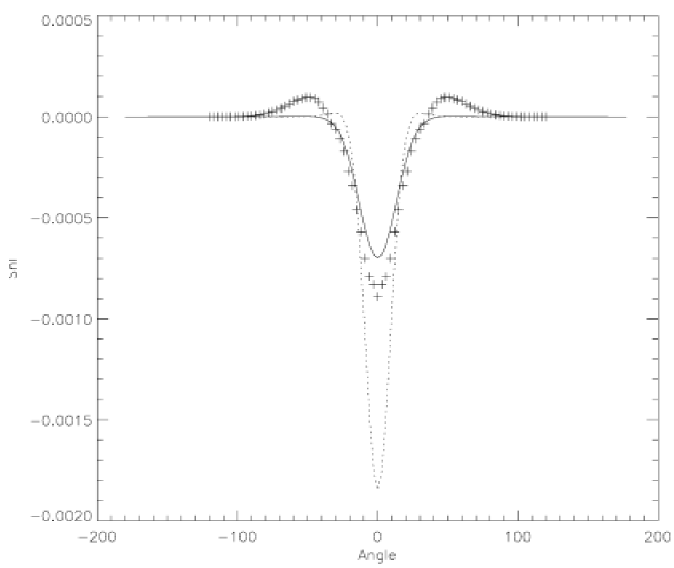

Figure 2. Angular dependence at fixed frequency $f = 0.16$. Crosses represent Resio - Tracy results, dashed line - first generation model, solid line - second generation model.

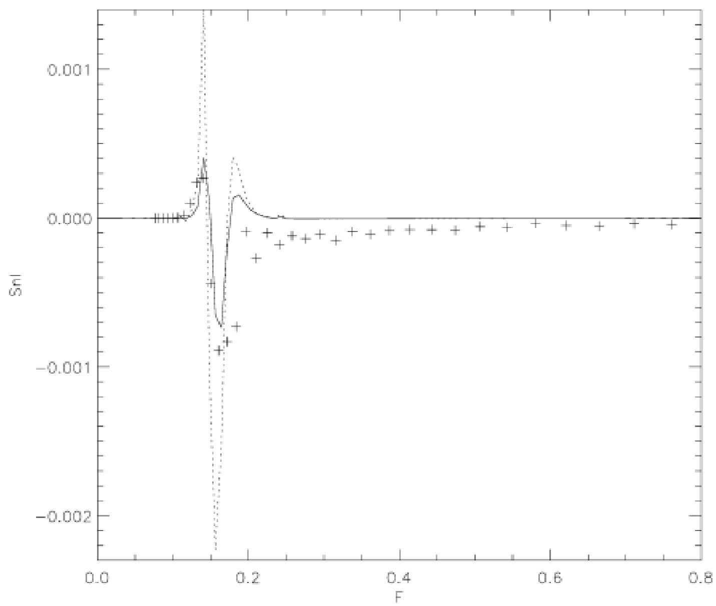

Figure 3. Nonlinear interaction term dependence on frequency at the angle $\varphi = 0$. Crosses represent Resio-Tracy results, dashed line -- first generation model, solid line - second generation model.

IMPACT/APPLICATION

The second generation diffusion model is the simple model which takes into account the effects of non-locality of the interaction of surface gravitational waves. As the first generation model, it preserves the constants of the motion, has righteous scaling and Kolmogorov solutions. Second generation model also contain first generation model as a special set of parameters.

Least square optimization allows to find unknown coefficients for the second generation model using the results of kinetic equation numerics. Comparison of the second generation model with kinetic equation numerics shows that second generation model improves directional and angular dependence of the spectrum with respect to first generation model. It still fails to describe fine details of the spectrum such as two - hump shape of the maximum and the high-frequency tail.

TRANSITIONS

Our first and second generation codes have been handed to people at the Army Water Experimental Station for testing.

RELATED PROJECTS

There are two related projects:

1. "The response of wind ripples to long-surface waves -- application to radar studies", ONR N00014-98-1-0439.

2."Statistical model for deep and shallow water waves", ARO DACA 39-99-C-0018.

REFERENCES

[1] Zakharov V. E. Stability of periodic waves of finite amplitudes on a surface of deep fluid*, J. Appl. Mech. Tech. Phys.*, 2, 190 - 198, 1968.

[2] Iroshnikov R. S., Possibility of a non-isotropic spectrum of wind waves by their weak nonlinear interaction, *Dokl.Acad.Nauk SSSR*, 280, 6, 1331-1325, 1985. (English transl. *Soviet Phys. Dokl*., 30, 126 - 128, 1985).

[3] Hasselmann S., Hasselmann K., Allender J.H. and Bernett T.P, Computations and parameterizations of the nonlinear energy transfer in gravity-wave spectrum. Part II*, J. Phys. Oceanography*, 15, 1378-1391, 1985.

[4] Zakharov V.E., Pushkarev A.N., Diffusion model of interacting gravity waves on the surface of deep liquid , *Nonlinear Processes in Geophysics*, 6, N 1 , 1999.

[5] Zakharov V.E., Pushkarev A.N., Presentation at the WISE meeting, Annapolis, 1999.

PUBLICATIONS

[1] Zakharov V. E., Pushkarev A. N., Diffusion model of interacting gravity waves on the surface of deep liquid, *Nonlinear Processes in Geophysics*, 6, N 1 , 1999.

[2] Zakharov V. E., Pushkarev A. N., Presentation at the WISE meeting, Annapolis, 1999.